



FFT Laboratory Experiment 3 Frequency Analysis of Periodic Signals

By: Michael W. Thompson, PhD. EE
Dept. of Electrical Engineering
Colorado State University
Fort Collins, CO

Purpose:

This experiment demonstrates the use of the FFT for analyzing the spectral content of a square wave and a triangle wave. Theoretical and experimental techniques are compared, and the trade offs between frequency resolution and aliasing are discussed.

Theory:

As noted in (reference 1.), the Fourier Transform of a square wave is closely related to its Fourier Series representation. In fact, the Fourier Transform of any periodic signal is given by

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi \alpha_k \delta(\omega - k\omega_0)$$

where ω_0 (rad/s) is the fundamental frequency of the periodic waveform, and the α_k 's are the (exponential form) Fourier Series coefficients.

A Fourier analysis of a square wave (peak=1, duty cycle = 50%) reveals that the magnitude of its series coefficients is given by

$$|\alpha_k| = \left| \frac{\sin(\pi k / 2)}{k\pi} \right|$$

Table 4.1 presents the Fourier Series coefficients (magnitude only) of a square wave and a triangle wave. This table is useful for determining the effective sampling rate for conducting an FFT analysis. For example, we might wish to neglect all of the spectral components after the 9th harmonic. Under this assumption we may choose a sampling rate of greater than $18 \omega_0$. However, one must bear in mind that the higher harmonics will "alias" and that care must be taken to avoid reaching misleading conclusions.

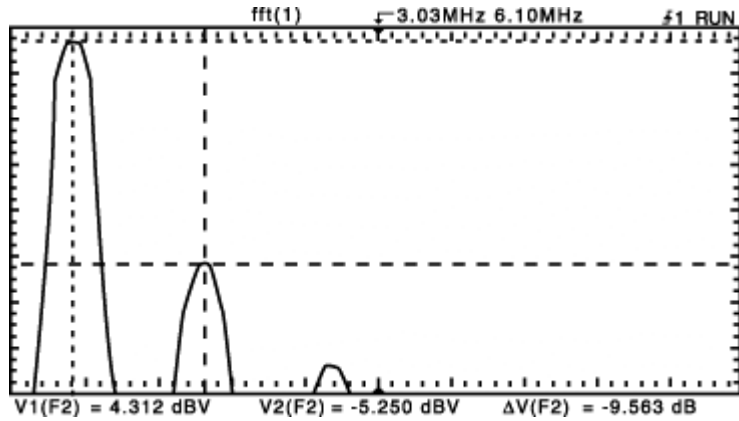


Square Wave		Triangle Wave	
Harmonic	Magnitude (dB)	Harmonic	Magnitude (dB)
1	+0	1	0
3	-9.54	3	-19.08
5	-13.98	5	-27.96
7	-16.90	7	-33.81
9	-19.09	9	-38.17
11	-20.83	11	-41.66
13	-22.28	13	-44.56

Table 4.1 Exponential Form Fourier Series coefficients.

Procedure:

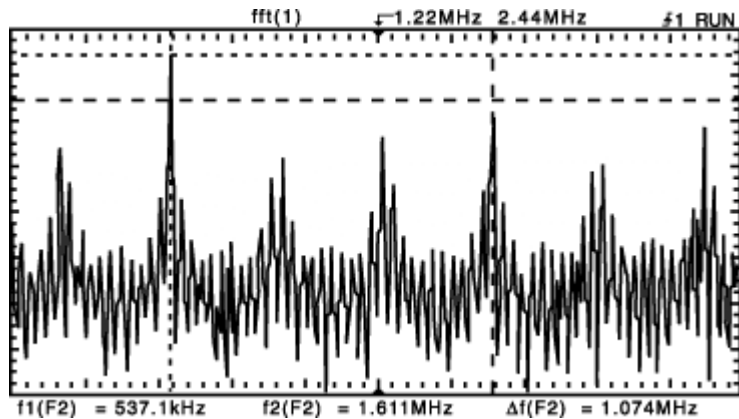
1. Use either the Agilent 54654A Training Kit circuit board (pin 2) or a function generator to produce a 500 kHz square wave (2 V peak). Use Autoscale to display the time-domain waveform.
2. Activate the "Functions" menu by selecting the ± key. Select On under the Function 2 portion of the display menu. Next, select 1 and turn off channel 1. At this point, the effective sampling rate is 200 MSa / s, and thus many of the square wave harmonics are displayed. Refer to Figure 4.1 for assistance with the FFT menu settings.
3. Now use the Time / Div setting to lower the effective sampling rate to 100 MSa / s. Use Figure 4.2 for assistance in setting the FFT menu selections. Use the Cursors and Find Peaks controls to measure the relative amplitude difference between the peaks. Compare these results to the theoretical values obtained by subtracting the magnitude values in Table 4. 1. For example, Table 4.1 indicates that there is a 9.54 dB difference between the magnitude of the first and third square wave harmonics. Figure 4.2 is designed to "zoom-in" on the first three harmonics. Notice that the flat top window provides the most accurate measurement of the relative amplitude between the harmonics.



Function2: Menu		
Effective Sampling Rate	Units/di	Ref Level
	v	
100 MSa/s	2.0 dB	5.00 dBV
FFT Menu		
Center Freq.	Freq. Span	Window
3.027 MHz	6.104 MHz	FlatTop

Figure 4.1

The 1024 point DFT (Magnitude) of a 2 V (peak) 500 kHz square wave using a Hanning window and an effective sampling rate of 200 MSa/s.



Function2: Menu		
Effective Sampling Rate	Units/div	Ref Level
5 MSa/s	10.00 dB	10.00 dBV
FFT Menu		
Center Freq.	Freq. Span	Window
1.255 MHz	2.441 MHz	Hanning

Figure 4.2

The FFT Menu settings for this figure are designed to isolate the first three harmonics of the square wave. The *Cursors* features of the oscilloscope are used to measure the relative amplitude difference (in dB) between harmonics.

- This step will show the effect of aliasing. Use the *Time / Div* control to select an effective sampling rate of 5 MSa / s. Refer to Figure 4.3 for assistance in selecting the FFT menu settings. In this display, the two largest peaks represent the 1st and 3rd square wave harmonics. However, as indicated by the figure, the higher harmonics are aliased and appear at lower frequency components in the FFT display.
- Repeat the above steps for a 1 V (peak) 500 kHz triangle wave.

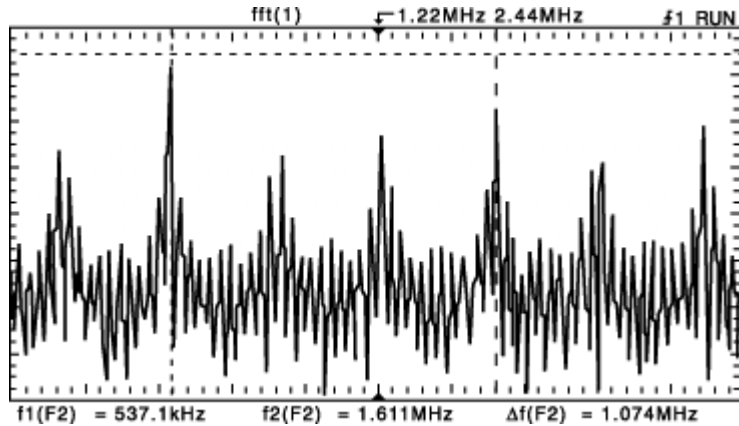
Questions

- Is it necessary to have a stable time-domain display in order to analyze the frequency content of a signal?
- Is prior knowledge of the signal's bandwidth required?
- Is it still possible to obtain useful information from the FFT display when components of the signal are aliased?



Conclusions

1. The flattop window is the most effective for measuring the relative amplitude difference between frequency harmonics.
2. Improper choice of the effective sampling rate can result in frequency domain aliasing whereby the higher harmonics appear as lower frequencies in the FFT display.



Function2: Menu		
Effective Sampling Rate	Units/div	Ref Level
5 MSa/s	10.00 dB	10.00 dBV
FFT Menu		
Center Freq.	Freq. Span	Window
1.255 MHz	2.441 MHz	Hanning

Figure 4.3

The 1024 point DFT (Magnitude) of a 2 V (peak) 500 kHz square wave sampled at 5 MSa/s. The cursors indicate the first and third harmonics of the square wave. The other peaks are due to the higher harmonics which, due to aliasing, appear as lower frequencies.

Answers to Questions

1. No, provided that the bandwidth of the signal is less than 488 kHz. Otherwise, the repetitive sampling scheme, which allows the oscilloscope to display time-domain signals with bandwidths greater than 488 kHz, requires a stable waveform.
2. To avoid aliasing we need to know the bandwidth of the input signal. One strategy is to initially choose a large sampling rate and then slowly decrease the sampling rate until aliasing becomes apparent.
3. Yes, but one must use caution. For example, in Figure 4.3 it is possible to use the aliased FFT display to locate the first and third harmonics in frequency with improved resolution.